Explanatory Indispensability and the Modal Perspective on Mathematics

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Abstract

Baker's Explanatory Indispensability argument maintains that mathematical objects are needed to *best explain* certain scientific facts. I argue that philosophers who reject mathematical objects but accept a certain (independently motivated) modal notion from the literature on potentialist set theory, can resist this argument by attractively nominalisticly paraphrase Baker's explanation in a way that avoids certain problems and limitations for existing Fieldian attempted nominalizations.

1 Introduction

Indispensability arguments maintain that (in one way or another) we cannot adequately make sense of current science without accepting the existence of mathematical objects. For example, Quine's classic indispensability argument holds that we need to quantify over mathematical objects to *literally state* our best scientific theories, and this commits us to the existence of such objects. And Baker's Explanatory (Enhanced) Indispensability argument maintains that mathematical objects are needed to *best explain* certain scientific facts[2].

In this paper, I'll argue that philosophers who reject mathematical objects but accept a certain (independently motivated) modal notion from the literature on potentialist set theory, can resist this argument. I'll show that we can use this notion to (uniformly) nominalisticly paraphrase a range of Platonist theories (along broadly the modal if-thenist lines suggested by Hellman in [14]). In particular, we can nominalize Baker's Platonist explanation in the central case he uses to argue that mathematical objects are explanatorily indespensable (accounting for Magicadas' prime length lifecyles). I'll note that my nominalistic paraphrase strategy avoids certain limitations and weaknesses of the influential Fieldian paraphrase strategy employed by Rizza in his existing nominalization of Baker's Magicadas explanation [21][10] (e.g., it can be applied Platonist theories invoking pure mathematical structures of arbitrary size). Then I'll argue that my nominalistic paraphrase for Baker's Platonist explanation of the Magicadas phenomenon is explanatorily as good as (indeed arguably better than) the original. However, two caveats should be noted.

First, I don't claim to fully answer Quinean or Explanatory indispensability arguments in this paper, as I don't claim to show my paraphrase strategy is applicable to *all* of our best scientific theories.

Second, my paraphrase strategy might not help philosophers who reject mathematical objects as part of a general physicalist project. For such philosophers may consider the key modal notion used in my paraphrases (a kind of logical possibility) inadequately physical. However, another reason for rejecting mathematical objects comes from considerations favoring a modal approach to mathematics. As Putnam noted in [19], (in many cases) we can seemingly take either a modal or a Platonistic approach to pure mathematics. But famous paradoxes in the philosophy of set theory (Burali-Forti paradox/indefinite extensability worries) seem to favor a modal approach to (pure higher) set theory, specifically a Potentialist understanding of set theory along the lines of [14, 16]. I aim to clarify whether Baker's Enhanced indispensability argument blocks taking a similarly modal view of all mathematics.

In §2 I will review the Explanatory indispensibility argument in the context of responses to classic Quinean indispensability arguments and Baker's influential Magicadas example. In §2.1 I'll review Rizza's Fieldian response to Baker's

Magicadas explanation, and discuss some limitations and problems for this approach. In §3 I'll introduce the key logical notions from potentialist set theory to be used in my paraphrases and then in §4 I'll use a simple example to motivate my uniform nominalistic paraphrase strategy. In §5 I'll state this uniform nominalistic paraphrase strategy. Then in §6.1 I'll show how to apply it to Baker's classic cicadas case and how it avoids the limitations on applicability noted for Field and Rizza above. In §7 I'll address a natural worry generated by the if-thenist form of my paraphrases. In §8 I'll conclude and note some important limitations to how far my nominalizaton strategy can be applied.

2 The Explanatory Indispensability Argument

As noted above, Quine proposed a classic indispensability argument along the following lines. We can't state our best scientific theories without quantifying over mathematical objects. And we should believe in the objects quantified over by our best scientific theories. So we should believe in mathematical objects.

Some nominalists, like Hartry Field, have answered this challenge head on, by rewriting scientific theories to avoid quantification over mathematical objects[10]. Others have rejected this demand for literal statement [5, 1, 24]. Drawing on scientists' use of idealized models and known falsehoods, like talk of infinitely deep oceans or continuous population functions, they say that it's OK if we can only evoke the scientific claims we believe are true by engaging in a fiction/pretense and saying things that are (literally) false. Accordingly, we can unproblematically quantify over mathematical objects in communicating our best scientific theories, even though no mathematical objects exist.

The Explanatory Indispensability argument strikes back at both kinds of nominalists: suggesting that mathematical objects are needed to **best explain** the phenomena accounted for in our scientific theories. Even if we don't need to believe in all objects quantified over in communicating our best scientific theories, the existence of mathematical objects is required to best explain the facts these theories explain. And even when we can literally state a nominalist theory which entails all the same consequences for non-mathematical objects as per Hartry Field, the resulting nominalist theory need not -and sometimes does not- provide an equally good explanation for these phenomena. So we have an inference to the best explanation for the existence of mathematical objects. As the Stanford Encyclopedia of philosophy puts it it [7]:

> [the] poster child for [arguments for the] explanatory indispensibility of mathematical objects is Baker's Magicadas explanation.
>
> North American Magicadas are found to have life cycles of 13 or 17 years. It is proposed by some biologists that there is an evolutionary advantage in having such prime-numbered life cycles. Prime-numbered life cycles mean that the Magicadas avoid competition, potential predators, and hybridisation. The idea is quite simple: because prime numbers have no non-trivial factors, there are very few other life cycles that can be synchronised with a prime-numbered life cycle. The Magicadas thus have an effective avoidance strategy that, under certain conditions, will be selected for. While the explanation being advanced involves biology (e.g., evolutionary theory, theories of competition and predation), a crucial part of the explanation comes from number theory, namely, the fundamental fact about prime numbers.

Looking more closely at the scientific proposal Baker cites[2, 13] lets us make this claim a little more specific. Suppose that (at some point in their evolutionary history) magicadas faced some kind of predators which also have a cyclical lifecycle, hibernating for every n years. And suppose there were a range of different (otherwise) biologically viable options for both predator and magicada life cycles that natural selection operates on. Then we can argue that one should expect to see magicadas with prime numbered life cycle lengths when the biologically viable options for magicada and predator life cycles are related as follows. For some number l, the 'biologically viable' options for the predator's life cycle are exactly those numbers of years p such that $2 \leq p \leq \frac{l}{2}$, while the biologically viable options for the life cycles durations of magicadas are exactly those numbers of years c within the following range $2 + \frac{l}{2} \leq c \leq l$.

Cicadas' fitness reflects the number of years cicadas are awake in which predators are not. So we might represent the fitness of cicadas with life cycle of length c facing predators with cycle length p, by considering the following fraction:

$1 - \frac{\#(\text{years cicadas and predators both alert})}{\#(\text{years where cicadas are alert})}$

The best case for the cicadas would be to have a cycle length that shares a common factor with the predator's cycle length but is offset (e.g., cicadas are awake during even years and predators during odd ones). But plausibly the offset can change quicker than the life cycle length, in such a way that this possibility can be ignored.

So we can assume that that cicadas and predators cycles overlap at some year. Then they must overlap again $c * p$ years later. We will consider value of the fraction above for years from 1 to $c * p$. For, as the author of the original scientific article reference by Baker and Rizza puts it,"Note that this yields an average valid for $t \to \infty$ because the process is periodic with period $c * p$."[13]. We can then prove that[1]

[1] To calculate the number of years in which cicadas and predators overlap between the first year after the original overlap and the $c * p$th year after it, note that these are the years with numbers divisible by both c and p, hence those divisible by the $\text{lcm}(c, p)$. How many of these such years fall in the relevant range from 1 to $c * p$? The number of years in the range that are mutltiples of $\text{lcm}(c, p)$ is $c * p/\text{lcm}(c, p)$, which is provably equal to $\text{gcd}(c, p)$.

And the number of years cicadas are alert during the the $c * p$ years is simply $c * p/c$, that is, p.

Cicada fitness: $1 - \frac{\#(\text{years cicadas and predators both alert})}{\#(\text{years where cicadas are alert})} = 1 - \frac{gcd(c,p)}{p}$

So if we hold fixed predatory cycle length p, we have cicada fitness varying with $1 - \gcd(c, p)$.

Predators' fitness corresponds to the fraction of total years during which predators are alert in which cicadas are also alert. So, by a directly analogous argument, when cicadas have cycle length c the fitness of predators with cycle length p is $\gcd(c, p)/c$. Hence, fixing a cicada cycle length c, it increases with $\gcd(c, p)$.

When the assumptions about biologically viable cicada and predator life cycle lengths above hold, we can show the following. There is a prime number in the range of biologically viable cicada cycle lengths. All prime options for cicada life cycle length (in years) c are stable in the following sense. Whatever cycle length p the predators have, c delivers optimal results in avoiding overlap with predators of cycle length p (as we have $\gcd(c, p) = 1$, the minimum possible value) and hence won't be selected against. In contrast, for every non-prime cycle length c' in the option set, there is some possible predator cycle length which achieves $gcd(c', p) > 1$, hence predators having some such cycle length would be selected for. But then there would be selection against this cicada cycle length c', as, e.g., any prime length life cycle would do better at avoid overlap with predators of length p better than c' does ($\gcd(c, p) = 1$).

2.1 Existing Nominalizations

In existing work[21], Rizza advocates the same kind of position I will, regarding the explanation above: that the above case presents a genuinely mathematical explanation for a scientific phenomenon, but not one that commits us to the existence of mathematical objects. He backs this up by providing a particular nominalistic mathematical paraphrase for Baker's Platonistic mathematical explanation of the above facts about Magicacdas.

Rizza (in effect) points out that we can reconstruct a version of the Platonist argument which only quantifies over some initial segment of natural numbers – then that we can reinterpret that quantification over numbers as quantification over time points in some evenly spaced sequence of years with a starting point. Using relations like congruence between temporal intervals 'there's as much time between a and b as between c and d', we can then define an analog to successor, plus, times etc on this sequence of years, creating a temporal structure (a sequence of points in time) that's isomorphic to the initial segment of the natural numbers. Thus, we can reformulate the above Platonist argument that (under relevant assumptions) we should expect to see cicadas with prime length life cycles by systematically replacing claims about this initial segment of the numbers (and mathematical relations on it) with corresponding claims about this initial segment of the years.

Now in order to state the cicada explanans and explanandum, we need to somehow make claims about cicada life cycles, and the biologically viable options for alternative cicada and predator life cycles. So a Platonist logical regimentation of the explanation might use relations

- 'species ... has a life cycle of length ..' between animal/species or populations of cicadas and numbers
- 'the biologically viable options for cicada/predator life cycles (the life cycles cicadas/predators could have if selection favored it) are exactly the numbers within the range.... To .. years'.

Rizza's formulation nominalistic paraphrases use analogous relations between animals/species and points in the finite sequence sequence of temporal points. (To make this feel natural, we might think of the first relation as meaning something like 'x has a life cycle with length such that if x emerged during the designated year 0 then it would next be disposed to awake in year y, and then to repeat the cycle and give birth to children who would').

In this way, Rizza argues that that we can dispense with mathematical objects in Baker's example, by giving the above nominalistic mathematical explanation instead.

Notably Rizza's nominalization strategy somewhat resembles Field's influential strategy for nominalizing physical magnitude claims in [10] (which I won't summarize the latter here). Both paraphrase strategies (in effect) assume the existence of a physical structure which resembles a mathematical structure used in in the theory to be paraphrased (a finite plurality temporal points isomorphic to initial segment of the natural numbers in one case and an infinite plurality of space time points isomorphic to the reals in the other case) and appeal to measurement theoretic uniqueness theorems to show that their paraphrase delivers correct truth values in all scenarios where the relevant physical assumption holds. One might argue Rizza's story has an advantage over Field's in requiring weaker physical assumptions, as Rizza only needs to assume there are a finite number of temporal points (but I will question whether the benefits of this are worth the cost below).

2.2 Weaknesses of existing paraphrases strategies

Rizza's paraphrase strategy (like Field's which inspired it) have limited generalizability because (as noted above) they both proceed by finding a copy of the mathematical structures mentioned by the Platonist theory they're trying to paraphrase in the physical world. This significantly limits how widely they can be applied.

For one might well want to appeal to larger mathematical structures when stating physical regularities or explaining why they hold. Consider the example of non-elementary proofs in mathematics. Sometimes the most illuminating proof of some fact about the real numbers involves considering them within the complex numbers. Similarly one might expect that much larger structures, e.g., segments of the hierarchy of sets could be relevant to giving an illuminating explanation for mathematical phenomena[2]. And as Baker pointed out in [3] even in cases where we can prove some science-relevant mathematical constraint on reality using relatively small mathematical structures, we can often prove a more powerful and general claim (and hence show that the law in question would hold under a wider range of cases[3]) by appealing to more varied and sometimes larger mathematical structures.

One might even argue that Rizza's paraphrase isn't as good an explanation as the Platonist's original because it fails to capture the full generality of Platonist explanation (and the mathematical facts behind it). Rizza shows can state and prove a suitable theorem for any particular value of L (and all suitably truncated versions of all needed lemmas which only talk about the initial segment). But without assuming there are an infinite number of spacetime points, he can't state (much less prove) the general theorem quoted for arbitrary values of L. One also might worry that Rizza's paraphrase strategy doesn't let us reconstruct the Platonist's justification for why facts about the gcd of the life cycles matter. For example, he can't paraphrase remarks like "Note that [a certain fraction] yields an average valid for $t \Rightarrow \infty$ because the process is periodic with period $[c * p]$." which the original paper uses in to argue that cicada fitness goes up as $\gcd(c, p)$ goes down.

I will show how proponents of a modal perspective on mathematics can use

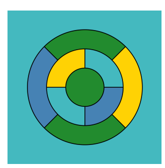
[2]As Fefferman notes in [9] appeal to the existence, (or at least logical possibility/coherence) of very large mathematical structures may provide our only reason for thinking that certain mathematical axioms, and hence figure indispensably in our best explanation for why no proofs inscriptions of certain kinds exist.

[3]That is, one can show that fewer physical assumptions are necessary to guarantee that the law applies.

re-use the logical notions needed to formulate potentialist set theory provide a different kind of nominalistic paraphrases which avoid the limitations noted above. For simplicity, I will formulate my paraphrases using the notion of conditional logical possibility from the streamlined formulation of potentialist set theory in [4]. However, it should be noted that the same formal work could be done using the second order relation quantification and actuality operator employed by [15] as [4] makes clear [4] and that the basic modal if-thenist paraphrase strategy I'm whose explanatory power and other virtues I argue for here is suggested in Putnam [19] and Hellman[15] (though not the particular details)
.

3 Conditional Logical Possibility

To introduce the key notion of conditional logical possibility , consider the following motivating example. Suppose we have a map like this:



It's logically impossible, given the facts about how 'is adjacent to' and 'is a country' apply to the countries on this map, that each country is either yellow, green or blue and no two adjacent countries are the same color.

$\Diamond_{R_1 \ldots R_n}$ Generalizes the notion of $\Diamond$. When evaluating logical possibility $\Diamond$ we: ignore all limits on the size of the universe. We consider only the most general combinatorial constraints on how any relations could apply to any ob-

[4]We could probably also do the same work by coding all nominalisticly acceptable objects and relations satisfying the definable supervenience conditions below with sets and then using the notions of interpretational possibility, the logically necessary essences of sets, and plural logic Linnebo uses to develop potentialist set theory in [17].

jects[5]. And we ignore subject matter specific and metaphysical constraints so, e.g., $\Diamond\exists x(Raven(x) \wedge Vegetable(x))$ comes out true.

When evaluating conditional logical possibility $\Diamond_{R_1...R_n}$ we do almost the same, but we also hold fixed the application of certain specific relations $R_1 \ldots R_n$.

I will use the notation $\Diamond_{R_1...R_n}$ to express claims about what's logically possible *given the facts about how certain relations apply.* Consider:

> **C&B**: 'It is logically impossible, given what cats and baskets there are, that each cat is sleeping in a basket and no two cats are sleeping in the same basket.'

There's an intuitive sense of 'logically impossible' on which this claim will be true iff *there are more cats than baskets* in the actual world. I'd write this as follows.

$\neg\Diamond_{cat,basket}$ [Each cat is sleeping in a basket and no two cats are sleeping on the same basket.]

Using this language of conditional logical possibility, we can express the non-three colourability claim above follows[6] :

[5]c.f. Frege on logic and absolute generality in [12].

[6]One can further explain and motivate my notion of conditional logical possibility by appeal to Stuart Shapiro's notion of structures in works like [22]. Shapiro introduces a notion of systems, consisting of some objects to which some relations $R_1...R_n$ apply, considered under some relations e.g., "An extended family is a system of people with blood and marital relationships, a chess configuration is a system of pieces under spatial and "possible move" relationships, a symphony is a system of tones under temporal and harmonic relationships, and a baseball defense is a collection of people with on-field spatial and "defensive-role" relations."

Then he says that a structures are 'the abstract form' of a system, which we get by "highlighting the interrelationships among the objects, and ignoring any features of them that do not affect how they relate to other objects in the system." Thus, for example, the natural-number structure will be exemplified by a number of different systems: the strings on a finite alphabet in lexical order, an infinite sequence of strokes, an infinite sequence of distinct moments of time, etc'

In these terms we might say that my notion of logical possibility given the facts about how certain relations apply is logical possibility given the fact about what structure is instantiated by the objects satisfying at least one of these relations (considered under these relations). That is:

It is logical possible, given the $R_1 \ldots R_n$ facts, that ϕ (i.e., $\Diamond_{R_1...R_n}$ iff some logically possible scenario makes ϕ true while holding fixed what structure (in Shapiro's sense) the system formed by the objects related by $R_1 \ldots R_n$ (considered under the relations $R_1 \ldots R_n$) instantiates.

$\neg\Diamond_{adjacent,country}$ Each country is either yellow, green or blue and no two adjacent countries are the same color.

Finally, in articulating potentialist set theory (and stating the potentialist paraphrases I'll advocate below) we will also want to make claims about the logical possibility or impossibility of claims which themselves employ the conditional logical possibility operator). That is, we can say things like the following. It would be logically possible for 'cat' and 'basket' to apply in such a way that it would be logically impossible, given what cats and baskets there are, for each cat to sleep on a different basket.

$\Diamond(\neg\Diamond_{cat,basket}$ Each cat is sleeping in a basket and no two cats are sleeping on the same basket.')

I take this claim to be true for the following reason. It's logically possible (holding fixed nothing) that there are 4 cats and 3 baskets. And relative to the scenario where there are 4 cats and 3 baskets, it's not logically possible, given what cats and baskets there are, that each cat slept on a basket and no two cats slept in the same basket.

4 Motivating Case: Three Colourability

To illustrate how this conditional logical possibility operator is useful for providing illuminating nominalistic mathematical explanations of physical phenomena – and why one might think these explanations *improve on* Platonistic ones – let's return to the case of three colourable maps.

Suppose that a certain map (perhaps one with infinitely many countries) has never actually been three colored. A good explanation for this fact might be that (in a mathematical sense) the map isn't three colorable.

A natural Platonistic explanation along these lines goes as follows.

> **Platonistic Non-Three-Colorability**: There is no function (in the sense of a set of ordered pairs) which takes countries on the map to numbers $\{1, 2, 3\}$ in a such a way that adjacent countries are always taken to distinct numbers.

However, we now have an additional nominalist version of this claim to consider.

> **Modal Non-Three-Colourability**: $\neg\Diamond_{adjacent,country}$ Each country is either yellow, green or blue and no two adjacent countries are the same color.

And the above modal explanation can seem to be at least as good, indeed better than the nominalist explanation.

In particular, one might argue that the Platonistic non-three-colourability principle only intuitively explains because we have background knowledge of a relationship between set theoretic facts and the modal facts above. Specifically, we think that that there are functions corresponding to *all possible ways* of pairing countries with one of the numbers 1, 2 or 3, and hence all possible ways of 'choosing' how to color these countries. For if we didn't accept this then we would have no reason to suppose that there really was a function corresponding to a potential 3-coloring[7].

Thus, it may seem that the real explanatory work here is being done by the modal principle; claims about what mathematical objects like set theoretic

[7] The argument I'm making here is quite similar to Field's argument that we need to have a notion of logical possibility that's distinct from having a set theoretic model, even though the completeness theorem (ultimately) winds up showing that the two notions are extensionally equivalent for first order claims.

functions exist witnessing facts about how it would be logically possible for any predicates to apply don't really add anything to the explanation.

Indeed, one might argue that the Platonist account only seems explanatory and satisfying because the modal facts (about conditional logical possibility) make us feel that we've explained the phenomenon. For we if imagine giving up the assumption that there are sets/functions corresponding to all logical possibilities for how colors could apply, then the Platonist story no longer feels explanatory. For we would no longer be able to infer from the fact that there's no function coding a way of three coloring the map that the map isn't and couldn't be three colored.

A Platonist might resist the above argument by saying that they get from set and function existence to the conclusion the map isn't three colorable in a different way. Platonists might say they that this inference is justified by appealing to something like the following non-modal comprehension schema – rather than to any modal notion like conditional logical possibility.

> Ur-element Comprehension Schema: For every English-definable predicate ϕ definable with parameters, if ϕ only applies to non-sets[8]:
>
> $$(\exists x)[set(x) \wedge (\forall y)(y \in x \leftrightarrow \phi(y))]$$

But note that the above schema only asserts that there are sets corresponding to every way that some predicates in our current language will actually apply to some objects. Thus it doesn't capture our intuitive idea the mere structure of how the countries are related by adjacency explains the fact that it has never been three coloured.

It also doesn't explain why we should expect it to be *physically and meta-*

[8]So, assuming certain popular axioms of set theory with ur-elements like that given in [18], only applies to set-many objects)

physically necessary that no intrinsic duplicates of the map are three colourable. And perhaps it also doesn't explain why we'd expect an analog of non-three colourability to hold for all triples of properties we might introduce via some 'logic preserving change to our language' that adds new predicates[9].

Accordingly, I think considering the above explanation provides a nice motivating example of the kind of thing that I'll be trying to do (and show I've done) more generally in this paper: provide a nominalistic-mathematical explanations for scientific facts which is as good (and in some senses even intuitively better than) Platonistic ones.

Interestingly, the modal nature of the nominalist paraphrase arguably matches ordinary language better than Platonistic paraphrases do. We tend to express the above thought about maps being three color*able*, rather than ontologically about maps *having three colourings*.

4.1 Nominalist Credentials

What about the nominalistic credentials of my modal notion?

First recall a nice argument of Hartry Field's for adopting a primitive modal notion of logical possibility simpliciter in [11]. He notes that we seem to have distinct notions of semantic entailment and syntactic derivability, as witnessed by the fact that Gödel's completeness theorem for first order logic isn't cognitively trivial, but rather an interesting mathematical fact that took some cleverness to prove. Prima facie we could either understand it set theoretically in terms of the existence of a model, or take it to be a modal primitive. However, there's a problem about understanding it set theoretically as follows. We think that if ϕ is logically necessary (i.e., semantically entailed by the empty assumptions) it has to be true. But if we interpret logical necessity in terms of the existence of set theoretic models, it's not clear what justifies this assumption. For we know

[9]See [18]

the whole world has a structure that's not witnessed by any set models (after all it contains the hierarchy of sets which is proper class sized), so why couldn't some statement which has no set theoretic model nonetheless actually be true? Field suggests that our situation is better understood by taking us to have a primitive modal notion of logical possibility, which we know to be sandwiched between syntactic consistency and having a set theoretic model in strength. When the completeness theorem showed that syntactic consistency were were coextensive for first order logical sentences, we could infer that all three notions were coextensive when applied to sentences in first-order logic (FOL).

Now, admittedly, one could think about conditional logical possibility (in particular, the sense in which conditional logical possibility claims hold fixed how certain properties apply) in a reifying way. For example, you could think that there's some abstract object called a structure and then analyze the above notion by saying (in the case above) that we freeze the facts about which structure the map instantiates. Thus, it might be tempting to think of them as showing that my modal notion is really hiding ontological commitments. However I think that such reifying notions are no more intrinsically clear or acceptable than the modal way of thinking about these facts that I've advocated (for example worries about *de re* possibility and quantifying in are well known).

I would draw the following moral from the above Fieldian argument about the concept of logical possibility simpliciter and consideration of the motivations for potentialist set theory (e.g. the Buralli-Forti paradox) that motivate potentialist set theory: just because you can reify a notion doesn't mean you should. When we see that mathematically/inferentially similar work can be done by either inflating our ontology or our ideology, we shouldn't always assume assume that the ontology inflating perspective is the right one. Taking the claims above to be nominalist doesn't violate the *letter* of Quine's criterion for ontological commitment (no quantification over anything other than countries is involved). And I think any sense in which it could be said to violate the *spirit* of Quine's criterion involves the kind of unjustified presumption in favor of expanding ontology rather than ideology, which I criticize above.

In the next section I deal with this concern by giving a uniform nominalist translation strategy which can be applied much more generally.

5 Nominalistic Paraphrase Strategy

5.1 The Basic Strategy

Now let's turn to the task of providing a general paraphrase strategy which addresses the point above. I'll suggest a general procedure T by which any Platonisticly acceptable sentence ϕ satisfying a certain **definable supervenience** condition can be turned into a nominalisticly acceptable sentence $T(\phi)$ which has the same non-mathematical content.

The basic idea behind my proposal is a familiar modal twist on 'if thenism' which has been developed in the case of pure mathematics by Putnam and Hellman[15, 19]. Roughly speaking, the idea will be that our nominalistic translation $T(\phi)$ of the platonist's ϕ says: it's logically necessary, fixing the facts about all relevant non-mathematical structures, that if there were also mathematical structures (or structures satisfying the platonist's categorical second order conceptions of these mathematical structures) then ϕ.

In order to implement this strategy we uniquely pin down the mathematical structures the Platonist talks in terms of, using only logical vocabulary (including the notion of conditional logical possibility above) and facts about non-mathematical structures? Intuitively, speaking, the **Definable Supervenience** condition says we can write a description D which thus 'uniquely pins down' all the structures the Platonist believes exist at each possible world w[10] in terms of their relation to the (fewer) structures the nominalist believes exist at possible world w.

So, for example, to translate a Platonist who believes in three types of mathemataical objects - natural numbers, sets of goats and partial functions from goats to natural numbers- it suffices to have a supervenience sentence D (call it D[numbers, goats-to-numbers functions]) which conjoins the following

- A categorical description of the natural numbers (i.e., a sentence which uniquely pins down how the Platonist thinks $\mathbb{N}, S, +$, apply, up to isomorphism)
- A description which pins down the structure of 'all possible' partial functions from goats to numbers[11], given the structure of the goats (and numbers)
- A collection of 'Julius Caesar sentences' specifying that the numbers are supposed to be distinct from the sets of goats, functions from goats to numbers and goats etc. [12].

When we have such a Definable Supervenience description D for a Platonist theory, the Platonist must agree this translation strategy $T(\phi)$ is 'extensionally adequate' in the following sense. The nominalistic sentence $T(\phi)$ is true at the correct set of metaphysically possible worlds (i.e., the worlds at which *they* take ϕ to be true), for every sentence ϕ with quantifiers explicitly restricted to range over (some finite list of kinds of) non-mathematical objects plus the mathematical structures specified by D.

[10] or all the ones they quantify over in stating their physical theories
[11] I will treat these as free standing mathematical objects
[12] This may include specifying that the numbers and sets of goats are distinct from all (the finitely) many types of non-mathematical objects relevant to the physical theory to be translated.

For note: the truth value of such a ϕ at each metaphysically possible world possible world will be completely determined by (structural facts about) how the platonist's mathematical and non-mathematical vocabulary applies at that world[13]. So if ϕ is true it will be logically necessary, given the relevant facts about how non-mathematical vocabulary applies (at that world), that if mathematical objects exist (as per D) then ϕ. That is, any logically possible scenario which preserves all the non-mathematical structures and has exactly the mathematical structures the Platonist believes must exist in such a scenario (as per D), will be one in which ϕ. And exactly the analogous point holds if ϕ is false.

Thus, when the Definable Supervenience Condition is satisfied, we can use the conditional logical possibility operator to write an if-thenist paraphrase of ϕ as follows. If $\mathcal{N}$ is a list of all nominalistic vocabulary used in D and ϕ (i.e., $\mathcal{N}$ is the list of all relations employed in these sentences which the Platonist and nominalist agree necessarily only apply to non-mathematical objects) then we have the following[14] :

$T(\phi) = \Box_{\mathcal{N}}(D \rightarrow \phi)$

Intuitively this says that it's logically necessary, given the structure of objects satisfying the nominalistic relations $\mathcal{N}$, that *if* there were (objects with the intended structure of) relevant mathematical objects then ϕ would be true. Note the Platonist must believe it is always logically possible to supplement the actual objects with objects that behave like the platonic objects and satisfying D, because they think such objects exist.

[13] I say structural facts because note that it doesn't matter which particular objects predicates and relations apply to doesn't effect the truth value of a sentence. Any interpretation of their vocabulary which is isomorphic to the intended interpretation will give ϕ the same truth value.

[14] In cases where we have a categorical description of the relevant structure (i.e., any two structures satisfying the description would have to be isomorphic to each other), this gives bivalent truth conditions for all pure mathematical statements. Note that when it's necessary to use second order quantification to pin down a categorical conception of the relevant structure, we can do this purely in the language of conditional logical possibility. This is demonstrated in [4]. Also note that the potentialist position I'm considering advocates a separate 'potentialist' (but still modal) treatment for unrestricted set theory.

To see how this plays out concretely, consider the following statement, which might be uttered by the Platonist consdered above

GOATS 'There are a prime number of goats.'

The Platonist will formalise this statement as follows.

ϕ_{GOATS} There's a $1-1$ function f[15], such that that maps the goats onto an initial segment of the natural numbers, ending below some prime number n.

And our nominalist can use the definable supervenience sentence D[numbers, goats-to-numbers functions] from above to translate the platonist's formalization of this English sentence into the nominalist sentence $T(\phi_{GOATS})$. Thus they can state the claim that there are a prime number of goats as follows.

$\Box_{goat}$[D[numbers, goats-to-numbers- functions] $\rightarrow \phi_{GOATS}$]

5.2 A Sample Supervienience Description

To get an even more concrete sense of what this looks like, note that there are only two things that can't be obviously formulated in first order logic in my description of the supervenience description D[numbers, goats-to-numbers-functions] above: the categorical description of the natural numbers, and the description of the partial functions from goats to numbers. In this section I will demonstrate a technique for creating such descriptions[16]

Let's start with categorically describing the numbers. We can categorically describe the natural numbers by taking the usual first order axioms of Peano Arithmetic and replacing the induction schema (call the result PA^{-}) with the following second order induction axiom[**?**][17].

$$(\forall X)\left[(X(0) \wedge (\forall n)\left(X(n) \rightarrow X(n+1)\right)) \rightarrow (\forall n)(X(n))\right]$$

[15] Here I treat functions as just another kind of mathematical object.

[16] See [4] for more on the power of this technique.

[17] I write 0 below for readability but recall that one can contextually define away all uses of 0 in a familiar Russellian fashion [**?**] in terms of only relational vocabulary

And we can reformulate this claim using conditional logical possibility, by picking any atomic predicate P[18] which isn't already used in our translation (say, 'is happy') and writing something like the following.

- **Induct**: '$\Box_{\mathbb{N},S}$ If 0 is happy and the successor of every happy number is happy then every number is happy.

In other words: it is logically necessary, given how $\mathbb{N}$ and S apply, then if 0 is happy and the successor of every happy number is happy then every number is happy.'

OK now what about the second half of our mission: pinning down the structure of partial functions from the goats to the numbers the Platonist believes in? We can nominalistically formalize this, using the same technique just demonstrated. Assume the Platonist theory has relations 'function()' and 'maps()' such that maps(f,x,y) iff f is a function that maps x to y, i.e., f(x)=y.

We can informally completely pin down the mathematical structure of partial functions from goats to numbers (henceforth just 'functions'), by saying two things:

- here are functions witnessing all possible ways of mapping some of the goats to some of the numbers[19].

- and there are no more functions than needed to this, i.e., every function maps only goats to numbers[20], and the functions are extensional[21].

The second claim is easy to formalise in FOL. And we can write the first using second order relation quantification as follows. For every relation R which

[18] Note that the different formalizations resulting from picking different P will be logically equivalent, and interderivable in a formal system for reasoning about logical possibility like [], because when evaluating conditional logical possibility only the arity of relations whose application isn't being held fixed matters

[19] Or in the limiting case of the partial function that's not defined anywhere, pairing no goats with numbers.

[20] That is, $(\forall x)(\forall y)f(x) = y \rightarrow$ goat(x)$\wedge$ number(y)])

[21] For every pair of distinct functions f and f' there's a goat such that f and f' map that goat to different numbers, or one of them maps it to a number and the other is undefined.

only relates goats to numbers[22] which is functional[23] corresponds to a function f. That is we can write:

$\forall R$[If R is functional and only relates goats to numbers then $(\exists x)$(function$(x)\wedge$ $(\forall y)(\forall x)$[maps$(x,y,z) \leftrightarrow R(y,z)$)]

To rewrite this using only first order language and the conditional logical possibility operator, we pick any two place relation that doesn't figure in the scientific theories we want to translate. For example, I will pick 'eucrastises', the relation x and y stand in when x restores y to the correct balance of humors (eucrasia).

I can assert that there are functions corresponding to all possible ways of mapping some goats to some numbers by saying the following. It's logically impossible given the structure of the goats, numbers and functions from goats to numbers, that 'eucratises only relates goats to numbers and applies functionally without there existing a corresponding function f [24]. Writing things in terms of necessity to parallel the structure above we have:. $\Box_{\mathbb{N},function,maps,goat}$[If 'eucratises' applies functionally and only relates goats to numbers then $(\exists x)$ (function(x) $\wedge(\forall y)(\forall x)$[maps$(x,y,z) \leftrightarrow$ eucratises(y,z))]

That is:

$\Box_{\mathbb{N},function,maps,goat}$[If nothing eucratises two distinct things and only goats eucratise and only numbers get eucratsed then $(\exists x)$ (function(x) $\wedge(\forall y)(\forall x)$[maps$(x,y,z) \leftrightarrow$ eucratises(y,z))]

So our total translation will have the following form[25].

$\Box_{goat()}[\psi_1 \wedge \Box_{\mathbb{N},S}(\psi_2) \wedge \Box_{\mathbb{N},function,maps,goat}(\psi_3) \rightarrow \phi_{GOATS}]$

[22] That is the sense that $(\forall x)(\forall y)Rxy \rightarrow$ x is a goat and y is a number])

[23] R is functional iff $(\forall x)(\forall y)(\forall z)[(Rxy \wedge Rxz) \rightarrow y = z]$

[24] Here we replace quantification over all second order two-place relations R, with claims about what's conditionally logically necessary, i.e. whatever must remain true for however our chosen two-place relation 'eucrastises' applies to the goats and functions structure the Platonist believes in.

[25] Here ψ_1 is the part of our descriptions of the numbers and functions from goats to numbers which is straightforwardly stateable in FOL.

Note that this nominalistic translation employs nested $\Box$s[26]. We describe the natural number structure the Platonist believes in modally, saying that the numbers are supposed to have a structure (when considered under the successor relation) that makes it impossible for 0 to be happy and the successor of every happy number to be happy without all numbers being happy. Compare this to the way we might describe the structure of a map by saying it's not three-colorable).

So our total modal if-thenist translation says: it's logically necessary, given structural facts about non-mathematical objects and relations, that if there are natural numbers satisfying the platonist's conception of them (i.e., if PA^- and the numbers' structure under the successor relation makes it impossible for 0 to be happy and the successor of every happy number to be happy without all numbers being happy) and... then ϕ_{GOATS}.

5.3 Avoiding Mathematical Vocabulary if Desired

Let me close this section by considering an objection to my basic nominalisation strategy. Some readers may worry about the above translations' use of mathematical vocabulary inside the $\Box$ of logical possibility. As stated, they talk about how it would be logically (not to say metaphysically!) possible for there to be natural numbers and sets with ur-elements. If you are a nominalist who thinks that 'number' is a meaningful predicate which just happens to necessarily not apply to anything (as Kripke argued for 'unicorn'), this is fine. However, those who don't like this option should note that, we could use any other first order predicates and relations that don't occur in the scientific sentence we want to translate instead. For example, we could uniformly replace 'number' and 'successor' in the translation above with,'angel' and 'is transubstantiated into' in

[26]So the point at the end of section ??, that nested conditional logical possibility operators freeze the scenario currently being talked about, not the actual world matters greatly.

our $T(\phi)$. This strategy follows Putnam's strategy for stating potentialist set theory in [19].

See appendix A for many more technical details on the above translations.

6 Advantages and Applicability

6.1 Advantages

We can immediately see how adopting the above strategy removes the limitations for Field and Rizza's strategies noted in §5. Those strategies couldn't mirror Platonist theories and explanations involving very large mathematical structures because they, in effect, depended on finding a copy of all mathematical structures employed by the Platonistic theory/explanation in the physical world. Thus, they couldn't translate Platonist theories quantifying over mathematical structures too large to have models in actual space and time. This raised doubts about the applicability of Fieldian paraphrase strategies, and their explanatory goodness (in comparison to Platonist alternatives) where they could be applied.

In contrast, my preferred paraphrase strategy has no problem applying Platonist theories that quantify over arbitrarily large mathematical structures (provided we have a suitable description of them). For it is logically possible that existing physical structures exist alongside arbitrarily large mathematical structures.

Thus, for example, unlike Rizza[], we have no problem saying that *for all* natural numbers L, if the biologically viable options for predator life cycles are those natural numbers p such that $2 \leq p \leq \frac{L}{2}$ and those for cicada life cycles are exactly those natural numbers c such that $2 + \frac{L}{2} \leq c \leq L$ and Magicadas have life cycles favored by the type of selection for fitness discussed above, they have life cycles lasting prime numbers of years. See appendix B for more details about how to apply the above paraphrase strategy to this case.

6.2 Colyvan's Worries

Adopting this paraphrase strategy may also let us address some concerns which Colyvan raises about the explanatory virtues of Field's paraphrases in[6]. Colyvan suggests that Platonist formulations of physical laws provide theoretical unification by letting us articulate the idea that two very different physical systems (say, a wave in water and an electromagnetic wave) have a similar physical structure and obey the same differential equation.

I take the point to be that a Platonist would say that both a wave in water and an electromagnetic wave can be described by a function which satisfies the same pure mathematical description (the differential equation). In this case my nominalist can say something similar, that it is logically possible for each physical structure to exist alongside a function capturing the relevant features of the physical system, and logically necessary that a certain shared description description (in this case the differential equation[27]) would be satisfied.

For example, in the case of a water wave, the Platonist would identify a function describing how the water's height at each location varies with time, (and say this function satisfies a certain differential equation). And my nominalist would say that it's logically necessary, holding fixed certain physical facts, that any function which captures the height of the water as a function of time (as specified in the relevant definable supervenience condition D[28]) obeys that same differential equation. Thus both Platonist and nominalist regimentations of our theories will make clear that the same description of a function's possible

[27] If you employ the strategy for removing mathematical vocabulary suggested at the end of the previous section, this description might be the result of uniformly substituting some other predicates/relations for mathematical predicates/relations in the Platonist's description.

[28] See appendix C for discussion of what this definable supervenience condition might look like

behavior characterizes the behavior of the water wave and the electromagnetic wave.

Similar considerations address another concern Colyvan raises in that chapter: that the Platonist can say what's correct about physical theories which get some mathematical equation right but the underlying physical structures which that equation describes wrong, and the nominalist can't. We can also use my nominalistic paraphrase strategy to say that two physical systems 'have shared structure' in the sense of being isomorphic (when considered under certain relations)[29]

7 A Worry about Instrumentalism

One might worry that my nominalization of the Magicadas Conditional above *isn't* as good at the Platonistic original, because it has an unappealingly instrumentalist form (as do all other uniform paraphrases produced by applying the strategy above). It might seem to resemble intuitively unattractive instrumentalist reformulations of scientific theories to avoid ontological commitment. For example, consider a version of our best actual physical theory (or Newton's) which says there is no moon and eliminates explanatory appeals to the moon, by positing suitably changed instrumentalist laws of gravitation, optics etc. This theory doesn't assert the existence of the moon, but says everything else will behave as it would if there were a moon with certain properties and our original physical laws applied.

The paraphrases associated with this moon-denying theory would be short,

[29]We can do this by applying my translation strategy T to a set theoretic formulation of this isomorphism claim, noting the point about definable supervenience of claims about layers of sets on facts about ur-elements. Or simply say: it is logically possible, holding fixed the $R_1, \ldots, R_n$ facts and the $S_1 \ldots, S_n$ facts that a relation F pairs objects satisfying at least one of the R relations with objects satisfying at least one of the S_i relations in a way that 'respects' these relations. That is, where R_i is a predicate we have for all x and y (which any of these relations apply to), if $F(x, y)$ then $R_i(x) \leftrightarrow S_i(x)$ and corresponding claims hold for relations of arbitrary arity.

like mine. And it makes the right predictions about everything that will happen to non-moon objects in the future. Yet there is intuitively a sense in which the actual existence of the moon does explanatory work in accounting for things like the motions of the tides, and the alternate moon denying theory which posits (seemingly ad hoc) variations in the laws of gravity near a certain point in the solar system seems to provide a worse explanation. Thus, although not indispensable to *stating* the constraints we expect to apply to the behavior of non-moon things, commitment to the existence of the moon very plausibly is indispensable to our best explanation of the behavior of non-moon things.

So a critic might wonder: how do we know that the nominalized Cicadas explanation just proposed (and all other explanations produced via the strategy outlined in §5) aren't bad in just that way? Isn't their form even suspiciously similar (both are broadly 'if-thenist')?

To address this worry, I will highlight an important point of disanalogy. To explain the motion of the tides, the moon-instrumentalist needs to posit controversial alternative physical laws. They must appeal to laws which are (intuitively) inelegant and less suitable to be supported by inductive generalization than the simpler theory which says that gravity works the same everywhere. Accordingly their overall theory strikes the Platonist as a priori less plausible than the moon endorsing theory (even though both imply all the same consequences for non-moon objects). And - whether or not one accepts the former claim- it is is certainly not something which moon advocates are already committed to accepting or take themselves to have strong independent reason to believe.

In contrast, in the Magicadas case (as we have seen), the nominalist theory which implies all the same data about concrete objects as our Platonist theory does is not only comparably plausible but actually something which the Platonist themselves *already accepts or has strong independent reason to believe*[30].

[30]More specifically, let A be (a Platonistic statement of) the contingent assumptions about

Thus, I claim, appeal to mathematical objects really is dispensable in Baker's cicada case. If my concept of conditional logical possibility and certain intuitive inference rules already have to be accepted and let us prove a nominalistic explanation for Magicadas' prime numbered life cycles which does all the unificatory work which the Platonist theory does (note that it has all the same inferential consequences), then it seems that the nominalist explanation can't be rejected as either implausible or unexplanatory by the nominalist. Thus, we don't have an inference to the best explanation from phenomena like the tendencies of certain Magicadas to have prime length life-cycles to the existence of mathematical objects.

8 Conclusion

In this paper I've argued that mathematical objects aren't needed to best explain Baker's Magicadas Phenomenon, or a range of other physical facts that are partly to be explained by mathematical facts. I noted that if we accept certain logical notions used in articulating potentialist set theory, we can nominalisticly paraphrase not only the sentences in Baker's magicada's phenomena this explanation (as Rizza already did in [21]) but a range of other mathematical explanations of scientific facts which cannot be nominalised by Rizza's Fieldian paraphrase strategy as well.

biologically viable cicada and predator life cyles and selective pressure needed for the cicada explanation. Let P be the conclusion that cicadas have a prime length cycle.

Then Baker's Platonist explanation for the cicadas fact appeals to the following law like claim: 'it's mathematically necessary that that if A then P'. Note that the point that this conditional is a law (not just a true material conditional) is necessary for the intuitive goodness of the explanation.

Using my nominalization strategy T yields an alternative explanation with the form T(A), 'it's logically necessary that if T(A) then T(P)' therefore T(P).

That is, we can replaced the Platonist's claimed mathematically necessary principle with logically necessary principle which the Platonist already accept (or can derive from intuitively good methods of reasoning about logical possibility like the axiom system proposed in REDACTED). This is an important point of disanalogy between my nominalist and the moon denier who replaces commitment to the laws of, say, Newtownian mechanics with commitment to gerrymandered looking laws of gravity and optics which moon accepters would reject.

I also defended the idea that my favored nominalist paraphrases are explanatorily on par with, or even superior to, the original Platonist explanation in various ways. Positively I argued that in a simple motivating case (non-three-colorability explanations) the modal perspective on mathematics seems to have an explanatory edge over the ontological perspective. And defensively I noted that my nominialized theories differed from paradigmatically bad instrumentalist physical theories in not appealing to any controversial new laws (but rather deriving the explananda from laws the Platonist already accepts in an equally general and illuminating and comparably concise way). I also argued that my paraphrase strategy lets us recognize structural analogies between physical systems just as well as the Platonist can, thus avoiding Colyvan's worries about Field's paraphrases.

A Translation Strategy Details

In this appendix I will explain more formally what my Definable Supervenience condition requires, and how to implement my translation strategy when it is satisfied.

A.1 Definable Supervenience

Informally we said the Definable Supervience condition required that we could specify what mathematical structures (and relations involving them) the Platonist takes there to be, in terms of facts about how some nominalistic relations (i.e., relations whose extension the Platonist and nominalist agree on) applies.

A.2 Nominalistic vs. Platonistic Vocabulary

A relation R counts as **nominalistic** vocabulary iff the Platonist and nominalist agree that it only applies to non-mathematical objects. So, for example, 'is a cat' and 'is taller than' are nominalistic relations. Platonistic vocabulary is all vocabulary that isn't nominalistic. So for example 'is a number', 'is an element of', 'is a set of goats', 'is a function from the cats to numbers' and '...has more than...fleas' are all Platonistic vocabulary.

In terms of these concepts, I will want to say that a list of Platonistic relations $\mathcal{P}$ (meant to describe the mathematical and applied mathematical objects) Definably Supervenes on a finite list of nominalistic relations $\mathcal{N}$ when there is a sentence D which the Platonist thinks holds at each possible world and completely specifies the behavior of the relations in $\mathcal{P}$ in that world (by specifying how the objects satisfying $\mathcal{P}$ relate to each other and to the objects satisfying $\mathcal{N}$). Note that this sentence, being finite, can only contain finitely many nominalistic relations.

When this definable supervenience condition is satisfied for some Platonistic vocabulary $\mathcal{P}$ we can translate every sentence ϕ such that for some list of nominalistic relations $\mathcal{N}$ all quantifiers in ϕ are restricted to objects which at least one of the relations in $\mathcal{P}, \mathcal{N}$ apply to[31]. This ensures the truth value of ϕ is completely determined by the 'structure' of objects satisfying the Platonistic and nominalistic relations. Note that, as one can categorically specify standard mathematical structures using conditional logical possibility[32], such structures automatically satisfy the definable supervenience condition.

[31] More formally, those objects which take part in some tuple satisfying one of these relations.
[32] See appendix A and [4]

A.3 Categoricity Over

Next, we want to express the idea that some description D 'specifies, for each possible world w, exactly what mathematical objects the Platonist thinks exist at w (and how all relevant Platonistic vocabulary applies)', so that D can be a suitable antecedent for our if then-ist translation.

First I will expand the notion of categoricity (all models of some theory are isomorphic) to the idea of being **categorical over** some list of relations. Informally, I will say that a description $D(N_1, \ldots, N_m, P_1, \ldots, P_n)$ is categorical for the $P_1, \ldots, P_n$ structure over the $N_1, \ldots, N_m$ structure if the facts about how relations $N_1, \ldots, N_m$ apply completely determine how $P_1, \ldots, P_n$ apply (and how they interact[33] with the relations $N_1, \ldots, N_m$).

For example, the following sentence D: SETS OF GOATS categorically describes how the Platonistic relations 'is a set of goats' and '...is an element of set of goats...' apply over the nominalistic relations 'is a goat'.

D: SETS OF GOATS

- The sets of goats is extensional[34].
- It's logically necessary, given the facts about how 'is a goat' 'is a set of goats' and '...is an element of set of goats...' are supposed to apply at any possible world, that if some goats are happy then there's a set of goats whose elements are exactly the happy goats.

We can generalize the above example as follows.

[33] In other words, if the sets of people, along with set membership, $(S_{\text{people}}, \in_{\text{people}})$ is categorical over the people P it's not just true that the number of sets of people is totally determined by what people exist but also facts such as whether or not any set of people is a person must also be determined.

[34] That is, sets of goats a and b are equal just if they have exactly the same members.

A.4 Definable Supervenience

Then we can state the definable supervenience condition as follows.

A list of relations $\mathcal{P}$ **Definably Supervenes** on a finite list of nominalistic relations $\mathcal{N}$ iff

- There's a sentence D (a 'Supervenience Description' which satisfies the following conditions (intuitively it must explain how the relevant Platonistic facts supervene on nominalistic facts)
  - D is formed using only relations in $\mathcal{P}, \mathcal{N}$ and all quantifiers in D are restricted to objects that satisfy at least one relation in this collection. The latter assumption ensures that the r 'only talk about' the structure of objects satisfying relations in P and N.
  - (From a Platonist POV) D is metaphysically necessary.
  - $\Box\Diamond_{\mathcal{N}}D$, i.e., the Platonist isn't supposing the existence of incoherent objects and indeed it's logically necessary that the $\mathcal{N}$ structure can be supplimented with Platonistic structure in the way that D requires. $\mathcal{P}, \mathcal{N}$
  - D is a categorical description of the $\mathcal{P}, \mathcal{N}$ structure over the $\mathcal{N}$ structure

Very many collections of Platonistic sentences involving pure mathematical structures (of reals, complex numbers etc..) and applied mathematical objects (of classes of physical objects, functions from physical objects to pure mathematical objects) straightforwardly satisfy this condition definable supervience condition. For example, note that the Platonist takes $D_{SetsofGoats}$ to be a metaphysically necessary truth. And $D_{SetsofGoats}$ specifies exactly what sets of goats there are at each metaphysically possible world w (and how the elementhood relation these sets of goats), given the facts about what goats there are at each world. Also, it's logically necessary that, however the goats are configured, they can be supplemented with sets as required by D sets of goats.

A.5 The Nominalist Paraphrase

With all these notions in place, we can finally define my proposed nominalist paraphrase strategy as follows.

When some Platonistic vocabulary $\mathcal{P}$ definably supervenes on some nominalistic vocabulary $\mathcal{N}$ via the Supervenience Description D then my **Nominalistic Translation T** will paraphrase every sentence ϕ where quantification is restricted to the objects in some tuple satisfying a relation in $\mathcal{P}$ or $\mathcal{N}$. The latter assumption ensures that ϕ only 'talks about' the $\mathcal{P}, \mathcal{N}$ structure. Thus, the truth-value of ϕ is determined by the behavior of the relations in $\mathcal{P}$ and $\mathcal{N}$, and unaffected by the behavior or the wider world around this structure.

$T(\phi) = \Box_{\mathcal{N}}(D \to \phi)$

B Magicadas

To apply the above strategy to Baker's Magicada case, we need to show that one can Platonistically formalize this theory in a way that satisfies the definable superveneince condition above. So we need a description that pins down all relevant Platonistic structures given the facts about how some nominalistic vocabulary applies[35].

We've already discussed how to do this for the natural numbers and, sets of temporal points, and functions from temporal points to numbers. But what about the Platonistic notions used to discuss (actual and biologically viable possible) life cycles?

[35] I take it that the Platonistic paraphrase I'll propose clearly satisfies requirement of being writable without unrestricted quantification in the sense of 5

- PlatonistActualLifecycle(x,n) iff an animal/species x has a life cycle of length n
- PlatonistPossibleLifecycle(x,n) it is a biologically viable option for animal/species x to have a life cycle of length n

These relations definably supervene on nominalistic relations of essentially the kind Rizza mentions. For example, a nominalistic version of the ActualLifecycle(x,n) might relate animals/species and pairs of temporal points.

- NominalistActualLifecycle(x,a,b) iff animal/species x is disposed to hibernate for the length of time between a and b and then repeat the cycle.

Specifically, we can uniquely specify how the relation ActualLifecycle (that the Platonist uses) behaves in terms of NomalistActualLifecycle, plus Platonist vocabulary concerning numbers and functions from numbers to years (which we've already shown satisfies the definably supervenience condition) plus a notion of temporal congruence[36] and temporal ordering, i.e., using the relations:

- TempCong(x,y,z,w) iff'as much time passes from x to y as from z to w')
- Before(x,y) 'temporal point x is before temporal point y'

Note that using the techniques for mimicking second order quantification above, we can categorically describe the natural number structure and uniquely pin down the intended structure of functions from these numbers to temporal points. We can then specify how the Platonist actual life cycle relation relates these 'numbers' to temporal points as follows.

An animal/species x bears the Platonistic 'actual life cycle length' relation to a natural number n iff x bears the *nominalistic* 'actual life cycle length' relation to a pair of temporal points a b, and there are n years between a and b. And

[36] This holds assuming that, like Rizza we have some definite description of a pair of temporal points picking out a canonical year.

(given the truth of the definable supervienience conditions for functions from numbers to years), this will be true if and only if some function counts off n temporal points separated by 1 year intervals with f(0)= a and f(n)=b.

PlatonistActualLifecycle(x,n) iff the usual definable superveneince conditions for the numbers and years is satisfied and there are temporal points a b such that NominalistActualLifecycle(x,a, b), and there's a function f which maps the numbers from 1 to n to temporal points in such a way that f(0)=a and f(n)=b and, for each number k, f(k) is before f(k+1), and the time between f(k) and f(k+1) is congruent to that between the beginning and endpoints of the canonical year.

C Physical Magnitudes

My paraphrase strategy only works when we can satisfy a certain Definable Supervenience condition above: when some principle the Platonist takes to be a metaphysically necessary truth uniquely specifies how all Platonistic vocabulary is supposed to apply, in terms of the facts about some finite collection of nominalsitic relations apply. And there's a longstanding line of worry, going back to [20], that (in some relevant sense) physical magnitude facts don't supervene on facts about any how finite collection of relations between non-mathematical objects apply .

The measurement theory results which Field and Rizza and others appeal to show that, if certain physical assumptions, hold then we can uniquely pin down the behavior of a length function by requiring that it respect the relations $\leq_L, \oplus_L$ where [37] the following pair of nominalistic relations (and assigns an appropriate value to your choice of unit).[38]

[37] I will say a function $l(x)$ respects $\leq_L, \oplus_L$ just if $a \leq_L b \iff l(a) \leq l(b)$ and $\oplus_L(a,b,c) \iff l(a)+l(b)=l(c)$.

[38] My presentation follows [23]

- $p_1 \leq_L p_2$ iff path p_1 is as long or longer than path p_2
- $\oplus_L(p_1, p_2, p_3)$ iff the combined lengths of path p_1 and p_2 together are equal to the length of path p_3

Specifically, we can prove the uniqueness claim above holds at all possible worlds where length is **Richly Instantiated** as per the following three principles.

Closure Under Multiples: Given a path x there are paths y with length equal to any finite multiple[39] of x.

Archimedian Assumption: No path is infinite in length with respect to another, i.e., if $x \leq_L y$ then some finite multiple of x is longer[40] than y.

Relational Properties: The relations $\leq_L, \oplus_L$ have the basic properties you would expect from their role as length comparisons[41].

That is, the assumptions above imply that there is a unique (up to multiplicative constant) length function (from paths to the real numbers) respecting $\leq_L, \oplus_L$.

If space necessarily satisfied the above assumption (or any variant that let one prove the same theorem), we would have the definable supervenience of the length function on finitely many nominalisticly acceptable facts (namely the facts about how the two nominalistic relations above apply).

But perhaps it is not metaphysically necessary that space satisfies any such assumptions. And, as [8] points out, it seems clear that other physical magnitudes like mass and temperature can - and do- apply in ways that aren't pinned

[39] Note that can use numbers and functions from paths to numbers (Platonistic vocabulary we've already seen definably supervenes on finite nominalistic vocabulary) to define 'x has length n times that of y' iff there are paths $c_i, 1 \leq i \leq n$ with $c_0 = x$, $c_1 = y$ and $c_i \oplus x = c_{i+1}$. So we can nominalisticly state this.

[40] Formally, anytime the length of a path a is less than the length of a path b there are paths $c_i, 1 < i \leq n$ with length i times that of a and c_n is longer than b. Again, logical possibility allows us to formalize this schema with an equivalent single sentence.

[41] For instance $\leq_L$ is transitive, reflexive etc. and $\oplus_L(p_1, p_2, p_3) \iff \oplus_L(p_2, p_1, p_3)$ etc..

down by analogous nominalistic mass/temperature relations even in close possible worlds.

So we might have the following worry. A Platonist regimentation of a theory can employ finitely many physical properties given by functions from objects to real numbers. But (one might think) no finite list of relations that hold between nominalisticly acceptable objects can suffice to fully specify the behavior of those functions at all (relevant) possible worlds. For this reason, one might fear that it will be impossible to write down a nominalistic sentence which is true at all the same metaphysically possible worlds as our best Platonist theory.

I won't try to solve this problem here, but will only make a brief suggestion which I develop in other work. When responding to classic Quinean and Enhanced indispensability worries, we plausibly only need to make sense of scientific theories *that we believe*. If it suffices to do this somewhat holistically and we believe that (as a matter of physical law) length is Richly Instantiated, then we can deal with the worry raised above as follows.

Suppose the Platonist worries that object masses (given by real numbers) can't be captured by any relations between nominalisticly acceptable objects. The nominalist can respond by invoking a four place relation M

- $M(p_1, p_2, m_1, m_2)$ which holds iff 'the mass of m_1 is as many times the mass of m_2 as the length of the path p_1 is to the length of the path p_2'.

Even though such a relation may not have any significance in physical law, we can see it suffices for our purposes. By the measurement theory results above, we can uniquely pin down the length function (up to a choice of unit) using nominalist relations $p_1 \leq_L p_2$ and $\oplus_L(p_1, p_2, p_3)$, all worlds where length is richly instantiated. At all such worlds we can then uniquely pin down the mass function (up to a choice of unit) by requiring that it respects the above four place relation between masses and lengths[42].

This allows us to offer a nominalistic translation of any Platonist theory ϕ (as it includes the claim that space is richly instantiated). For we can state the claim that space is Richly Instantiated in a formal nominalistic, as per the footnotes above. So if we can find a D which uniquely pins down the application of all Platonistic vocabulary at possible worlds *where Space is Richly Instantiated*, we can write the following nominalistic sentence will be true at exactly the set of possible worlds at which (the Platonist thinks) ϕ is true.

$N(\phi)$: Space is Richly Instantiated $\wedge T(\phi)$ where $T(\phi) = \Box_{\mathcal{N}}(D \rightarrow \phi)$,

But we have just shown how to do this for the disputed piece of Platonist vocabulary: the mass function. This strategy can be extended to handle more complex properties and holistically paraphrase theories making various different assumptions about space.

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[42] The assumption that length is richly instantiated also implies that, for any ratio r that might hold between the masses of x and y, there will be pairs of distances whose ratios come arbitrarily close to r from above and below, with the result that honoring the above constraint suffices to pin down the point above.

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