Multiverse Set Theory and Mathematical Explanations for Physical Facts

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Outline

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[inc: In this talk, I'll raise a worry about whether/how fans of multiverse set theorists can make sense of certain tractarian-flavored physical explanatory hypotheses.]

On a conventional understanding of set theory,

- There's a unique intended hierarchy of sets that contains, at each layer, sets corresponding to 'all possible ways of choosing' sets from lower levels.
- This determines a unique intended right answer to all set-theoretic questions, like the continuum hypothesis (CH), whose truth value only depends on the width of the hierarchy of sets.

Width Multiversism

In contrast, what I'll call **Width Multiverse** approaches to set theory (like[?]) propose that there are

- Multiple hierarchies of sets ("set theoretic universes") exist, making up the Set Theoretic Multiverse.
- There's no *intended hierarchy of sets* which contains 'all possible subsets' of the sets it contains
- Rather, for each universe V, there's a wider universe V[G] (got by taking a forcing extension) which adds sets to V, including a new subset G of some infinite set in V.

Accordingly, multiverse views combine

- ontological realism: sets exist
- truthvalue anti-realism: there's no
  - unique intended hierarchy of sets (even up to width)
  - generally intended right answer – beyond mathematicians' temporary choice to study one kind of a structure rather than another – to questions which depend on width and can be changed by forcing (like CH).

Hamkins' influential version of multiverse theory goes further with pluralism (and associated truthvalue antirealism), adopting the following principles:

- Countability Principle: Every universe looks countable from the point of view of some larger universe .
- Well-Foundedness Mirage Every universe's copy of the ordinals looks ill-founded from the point of view of some larger universe .
  - so there's there's no intended model of the numbers.

In this talk I'll explore a kind of explanatory indispensability challenge for multiverse theorists like Hamkins

- arising from an interesting kind of Tractarian-flavored logico-mathematical explanation for physical facts

Lazy Explanatory indispensability worry in a nutshell

- We're open to physical explanatory hypotheses by appeal to all possible ways of choosing (conventionally stated using sets)
  - e.g. 'that (physical) map is not three colored because there's no set coding a three coloring'
- But accepting width multiverse theory seems to require rejecting this notion of 'all possible ways of choosing' (or at least denying traditional claims about how it connects to set theory).
- Thus, multiverism threatens to require dismissing seemingly cogent physical explanatory hypotheses a priori.

In this talk, I'll

- introduce the idea of a lazy explanatory indispensability argument
- clarify the particular kind of explanatory hypothesis and corresponding lazy explanatory indispensability argument sketched above
- explore three possible ways of responding to this challenge

Unlike classic indispensability arguments (Quine, Baker and Colyvan) the lazy explanatory indispensability argument I'll develop

- attacks *truth value* anti-realism about math, not nominalism.
- says say we need some doctrine to adequately formulate
  - a seemingly cogent *possible* explanatory hypothesis (that is intuitively true at some possible worlds)
  - rather than some part of our actual best scientific theory (our actual best explanation for some phenomena)

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To motivate the idea of a lazy explanatory indispensability argument ...

- Consider Baker's classic indispensability argument (that certain cicadas have prime length life cycles in a way that's plausibly best explained by reasoning about primes minimizing overlap with cyclical predator population spikes)
- Imagine if the empirical facts about cicadas cited *turned out to be a hoax*.
- Many would still feel Baker's story presented a prima facie challenge for nominalism, in this case.
- For they'd be hesitant to accept philosophy of mathematics that required us to stop considering such explanations as a live option

Thus, plausibly, what I'll call a lazy explanatory indispensability argument for ontological realism would remain.

The core worry

Now let's turn to the specific kind of mathematical explanation for physical facts that drives my worry.

From a traditional POV, a favored notion of all 'possible ways of choosing' connects

- set theory
- constraints on non-mathematical objects

by having close close a priori connections to each, as follows.

- Facts about 'all possible ways of choosing' help determine a unique intended structure for the set-theoretic universe (up to width).
  - Each layer of the iterative hierarchy of sets is supposed to contain sets corresponding to all possible ways of choosing some sets occurring at lower levels (and/or ur-elements).
- Facts about all possible ways of choosing impose **counterfactual supporting constraints** on non-mathematical reality, in a way that can help predict and explain regularities involving physical objects...

For example, imagine a physical scenario where a (finite or infinite) physical map has never been three-colored, despite many changes in the colors of individual map regions.
[insert AI picture of a map stretching off to the horizon]

From a traditional POV it might be that the map is not three colored because it is three color*able* in the following sense

> ***Non-Three Colorability Hypothesis***: *The map isn't 3-colored because*
>
> - *there's no set coding a three coloring function in the hierarchy of sets with ur-elements*
> - *and this reflects the modal fact that there no possible way of choosing colors for the map regions (while preserving structural facts about how 'map region' and 'adjacent' apply) which three colors the map.*

And note that if the above (seemingly intelligible) hypothesis is correct, we'd expect that

- the map isn't actually three-colored.
- the map couldn't 'easily' have been three-colored
  - (i.e., it isn't three-colored at any close possible worlds, if these don't change structural facts about what tiles exist or adjacency relations).
- three-coloring is prevented by general, subject matter neutral logico-combiantorial constraints which also prevent it from being three-scented or three-textured etc.

I think these kinds of explanations [(logico-combinatorial explanations of physical facts, usually stated in set theory with ur-elements)] have various independently interesting features.

For example, they are somewhat Tractarian in flavor, insofar as they

- suggests that a combination of
  - general subject matter neutral combinatorial constraints (which treat all n place relations alike)
  - (structural) facts about about how 'tile' and 'adjacent' apply

  rule out the map being three colored

However (returning to our current argument) I submit that the non-three coloring explanation above is intuitively

- a cogent explanatory hypotheses, which we should be willing to consider, in response to some evidence
- a genuine metaphysical possibility

But what can the multiverse theorist say about this kind of explanation?

Width multiverse theorists deny there's a universe V containing, at each level α, sets corresponding to all possible ways of choosing elements from things (sets and ur-elements) at lower levels.

- For they say that, e.g., each universe V in the multiverse has a forcing extension V[G] that adds 'missing' subsets of copy of the natural numbers in V.

But

- if we aren't guaranteed that the hierarchy of sets contains sets witnessing 'all possible way of choosing' from ur-elements
- why should the claim that *there's no set coding* a three coloring of the map give us reason to believe the map isn't actually three colored (much less that it couldn't easily have been three colored)?

Note: this challenge arises even when all universes agree on the set existence claims used in the traditional explanation. For

- once you demolish the traditional bridge between set theory and non-mathematical reality
  - i.e., the assumption that a favored hierarchy of sets witnesses 'all possible ways of choosing' objects at lower levels
- it becomes prima facie unclear why the non-existence of certain sets (e.g. sets coding a three coloring) — in a universe, or the multiverse as a whole — should imply *anything* about how physical properties can apply to physical objects.

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I will now consider three families of ways a multiverse theorist could try to address the above challenge.

As noted above, multiverse theorists deny that there's a hierarchy of sets which contains all possible subsets of sets it contains.

- For they say that each universe V in the multiverse has a forcing extension V[G] that adds subsets of copy of the natural numbers in V.

So they must either

- Reject the notion of 'all possible ways of choosing' and replace physical explanatoray appeals to it with something else
- Accept this notion but say that (for some reason) no single hierarchy of sets can contain all possible subsets of sets it contains
  - e.g. only the multiverse as a whole witnesses all possible ways of choosing
- Accept this notion and somehow directly use it to give physical explanations, but deny it has any special connections to set theory (odd)

I will consider two possibilities of the first kind, and one of the second

Op 1:Constraints on All Sufficiently Physically Definable Properties

First, the multiverse theorist could

- replace appeals to 'all possible ways of choosing'
  - witnessed by the unique intended universe V
- with appeals to a notion physical necessity/law
  - witnessed by some favored universe V_p containing sets witnessing all physically possible ways for n-place relations to apply to ur-elements.

That is, they could say there's a *physically special* universe of sets with ur-elements V_p such that

- Even though V_p does not include sets corresponding to all (logically/combinatorially) possible ways of choosing some sets it contains (since, as per multiverse theory, it has an extension $V_p[G]$) [inc?]
- It's a physical law that V_p contains **all** 'sufficiently physically definable' subsets of sets it contains
  - i.e. physical law prevents properties from applying to actual physical objects in a way that isn't witnessed by the existence of an actual set of objects in V_p
- **(maybe)** V_p contains **only** the sets corresponding to physically possible outcomes of scenarios we'd describe as independent random events.

So their explanations might look like the following.

1. There's a certain physically preferred set universe V_p within the multiverse, which reflects lawlike constraints on how *all physically definable properties* can apply to actually existing objects (in the sense above)
2. There is no set witnessing a way of three-coloring the map in this physically preferred V_p.
3. Therefore the map isn't three-colored – and, indeed, it would be physically impossible for it to be three-colored (while facts about how map tiles are related by adjacency are held fixed)

But the kind of physical law this explanation posits can seem mysterious/a priori implausible.

How does physics control the outcomes of seemingly random independent events like coin tosses, to avoid 'realizing' (i.e., letting us use physical vocabulary to define) a missing subset?

- The notion of 'suitably physically defined' properties can seem too gerrymandered to figure in a brute/fundamental physical law.
  - c.f. intuitions that 'observation collapses the wave function' couldn't be a law because observation is the wrong kind of concept to occur in a fundamental physical law
- Maybe one could formulate some other fundamental law in more elegant terms which (given plausible metasemantic assumptions) implies this constraint on intuitively physically definable notions?

Alternatively

- If we're confident that the multiverse contains some universe witnessing all physically realized patterns..
  - (i.e. this universe contains sets corresponding to all ways physically definable properties actually apply)
- we could stipulate that, when talking about physical objects, we're always to be understood as talking about such a universe

  - so the non-existence of a set coding a three coloring will imply the map is not actually three colored.

But this strategy seems even worse. For in this case

- Naive set theoretic explanations like, 'The map won't be three colored because it's not three colorable' won't have **counterfactual supporting law-like** force.
- Claims about the non-existence of a three coloring function will only provide a dormative virtue non-explanation for physical regularities. Compare
  - Jake doesn't know how to drive because his name isn't on the list of people who know how to drive.
  - 'The map won't ever be three colored because a hierarchy of sets V that contains set extensions for all sufficiently physical properties doesn't contain a three coloring function/sets giving suitable extensions for 'red', 'green' or 'blue'.

Opt 2: FOL Surrogates

Next, (in the specific case above) one might try to replace appeals to all possible ways of choosing with plain first order logical deductions from non-mathematical facts.

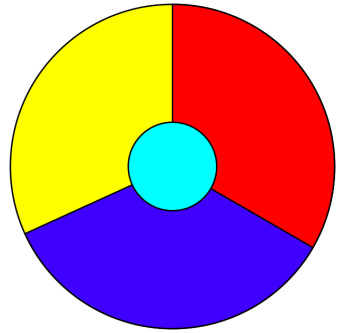
- I'll discuss two ways of realizing this approach, noting specific worries for each
- and then raise a major objection to both

Consider our sample explanation again

- 'That map was never three colored, because it is not three colorable (i.e., there is no three coloring function)'

Strat 2A: Replace traditional set theoretic explanation with a map-specific deduction proof the map is not three colored from FOL facts about the adjacency relations between tiles on the map.

- For example, suppose that part of the infinite map in question looks like this...



Then we might replace appeal to set models with (a FOL to formalalized version of) the following reasoning:

> *There are three tiles (the outer ring tiles) each pair of them touches. Hence these tiles must each be given distinct colors.*
>
> *There's another tile, (the center tile) which touches each of these three ring tiles and hence must have a fourth tile. Thus the tiles cannot each be red, green or blue, without some two adjacent tiles having the same color.*

This kind of substitution can always be done.

- i.e. whenever a map isn't three colorable, there will be some some finite collection of FOL facts about tiles and adjacency imply that it isn't three colored[1]

[1] Imagine an infinite language with a name for each country, and a theory that lists how all countries are related. By completeness, this theory FOL implies that the map isn't three colored. So by compactness, a finite fragment of T implies the map isn't three colored.

However

- The resulting deduction from specific facts won't be as unifying/explanatory as the original
  - c.f. Putnam on high level explanations for why a square peg won't fit through a round hole
- This kind of substitute explanation can only be given when we know specific facts about the specific map (which imply that it isn't three colored).
  - But we can (rationally) entertain the original hypothesis in cases where we don't know specific facts allowing such a proof.

FOL Surrogates

Strat 2B: Replace the traditional explanation with a claims about FOL about *derivability*[2]

- e.g., "Maybe the map hasn't ever been three colored because *there are some* truths about map regions and adjacency which FOL imply that it's not three colored'."

[2]Happily forcing doesn't change the natural numbers in V, hence won't change facts about what your background set theory thinks is derivable.

FOL Surrogates

Problems:

- This strategy's appeal to a (realist, unanalyzed) notion of derivability combines awkwardly with Hamkins' Multiverse proposal
  - for Hamkins rejects an intended model of the natural numbers.
  - and the notion of derivability is closely tied to of the existence (in the intended model) of numbers Gödel coding a derivation.
    - e.g. if there are primative objective facts about derivability, shouldn't we at least say that all models that 'get derivability wrong' in the sense of Gödel coding are objectively unintended?
- Maybe multiversists who accept an intended model of the numbers would find it more congenial, but ...

There's a bigger problem which applies to both versions of the strategy in this section:

- The strategy seemingly can't be applied to more complex - but equally seemingly cogent - examples of mathematical explanatory hypotheses.

For example, here is another seemingly cogent kind of explanatory hypotheses (that we'd intuitively like to allow room for) with a more complex structure

For example, consider physical explanatory hypotheses with a $\forall\exists$ rather than $\neg\exists$ structure.

> ***Troop Distribution***: *The reason why no one has succeeded in holding such-and-such map region is that, for every possible way of stationing defending troops in countries on the map satisfying ... constraints, there's a way of stationing attacking troops such that ...*

This doesn't correspond to claims about some non-mathematical fact being FOL derivable from another in any obvious way.

- so the above strategy can't be applied

Op 3: Invoke Whole Multiverse

Finally you could say that facts about 'all possible ways of choosing' are witnessed by facts about what sets exist within the multiverse as a whole.

- e.g. 'The map isn't three colorable' = there are no sets x, y, z anywhere in the multiverse (of hierarchies of sets with ur-elements) such that every map tile is an element of either x, y, or z and no adjacent map tiles are both in x, both in y, both in z.

But this seems contrary to the spirit of the Hamkins' multiverse (as I understand it).

- If (as I gather Hamkins thinks) we can't quantify over all sets anywhere in the multiverse when doing pure mathematics, why can we do this when doing science?
- If we can quantify over all sets everywhere in the multiverse when doing applications (and these witness all possible ways of choosing as required by this strategy)...
  - then we have (in effect) second order quantification over ur-elements
  - So we can say some physical objects form an intended model of the natural numbers
    - (e.g. the stars satisfy PA_2 with 'farther from earth than' playing the role of $>$).
  - And (use this description to give) if-thenist paraphrases of number claims, pinning down intended realist truth-values for claims about the numbers.

Similarly adopting this strategy threatens to undermine sntirealism about CH

- Suppose we say can capture physical explanations which talk about all possible ways of pairing up physical objects (i.e., simulate second order function quantification over ur-elements)
  - to formulate intuitively cogent explanations that appeal to all possible ways of adding 1-way bus service between certain towns
  - because the whole multiverse contains sets corresponding to all possible ways of choosing some ordered pairs of ur-elements.
- Then we can use the same device to describe different kinds of physical objects as having (respectively) the intended structure of the natural numbers, the reals, and something strictly in between in size.
  - so that CH is intuitively true iff it is metaphysically possible for this description to satisfied

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In this talk I've

- highlighted a kind of intuitively cogent tractarian-flavored physical explanatory hypotheses
- argued that width multiverse theorists (and all who reject a favored notion of 'all possible ways of choosing') face a challenge about how to understand/formulate/replace such explanation

I've discussed three possible replies for the multiverse theorist

- highlighting problems for each

Even if you don't care about multiverse theories particularly, I think thus considering these difficulties highlights a specifc interesting way that

- doctrines in seemingly pure philosophy of mathematics (multiversism, truthvalue antirealism)
- have fairly direct impliciations for (or at least raise questions about)
  - notions of logical possibility 'all possible ways of choosing'
  - the space of legitimate physical explanatory hypotheses

Bibliography